Written Exam Economics Summer 2020

Advanced Financial and Macro Econometrics

June 16, 10:00 – 22:00

This exam question consists of 5 pages in total

Answers only in English. A take-home exam paper cannot exceed 10 pages (plus a few pages with additional estimation output). One page is defined as 2400 keystrokes.

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Be careful not to cheat at exams!
Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

You can read more about the rules on exam cheating on your Study Site and in part 4.12 of the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

1 Bootstrap

#1.1 Time Series with Missing Observations

Consider an AR(1) process given by

$$y_t = \rho y_{t-1} + \varepsilon_t, \tag{1.1}$$

for $t = 1, 2, ..., T$ with $T = 2N$ an even number. Moreover, for the statistical analysis y_0 is fixed, and $\{\varepsilon_t\}_{t=1}^T$ are assumed to be i.i.d.N$(0, \sigma_\varepsilon^2)$ distributed. Consider a setting where only $(y_0, y_2, y_4, ..., y_T)$ are observed while $(y_1, y_3, y_5, ..., y_{T-1})$ are unobserved. Set

$$(x_0, x_1, ..., x_n, ..., x_N) = \left(y_0, y_2, ..., y_{t/2}, ..., y_T\right), \tag{1.2}$$

with $N = T/2$.

(a) Show that x_n can be written as an AR(1) process in terms of an autoregressive parameter ϕ and innovations sequence $\{\eta_n\}_{n=1}^N$. Specify ϕ and η_n in terms of ρ and ε_t.

#1.2 Likelihood Ratio Test

Based on the AR(1) model for $(x_0, x_1, ..., x_n, ..., x_N)$ let $\hat{\phi}_N$ denote the MLE of ϕ and $\mathrm{V}(\eta_n) = \sigma_\eta^2$. Consider the hypothesis given by

$$H : \phi = f, \tag{1.3}$$

for some fixed f, with $-1 < f < 1$, and the Wald statistic

$$W_N = (\hat{\phi}_N - f)^2 \sum_{n=1}^N x_{n-1}^2 / \sigma_\eta^2. \tag{1.4}$$

(a) Show that $W_N \xrightarrow{D} \chi_1^2$ as $N \to \infty$. Be explicit about your assumptions.

#1.3 Wild Bootstrap Scheme

Consider a bootstrap scheme defined by

$$x_n^* = f x_{n-1}^* + \eta_n^*, \tag{1.5}$$

with $x_0^* = x_0$ and $\eta_n^* = w_n \hat{\eta}_n$, where $\{w_n\}_{n=1}^N$ are i.i.d.N$(0, 1)$ and $\{\hat{\eta}_n\}_{n=1}^N$ are given by

$$\hat{\eta}_n = x_n - f x_{n-1}. \tag{1.6}$$

(a) What is the distribution of x_n^* conditional on x_{n-1}^* and the original data $\{x_n\}_{n=0}^N$? Argue in particular that $\mathrm{E}^*(x_n^*) = 0$ if $x_0 = 0$.

#1.4 Bootstrap Statistic

Consider

$$M_N^* = N^{-1/2} \sum_{n=1}^{N} m_n^*, \quad \text{with} \quad m_n^* = \eta_n^* x_{n-1}^*. \tag{1.7}$$

(a) Argue that under the hypothesis, H, m_n^* is a MGD sequence.

(b) Next use that,

$$N^{-1} \sum_{n=1}^{N} x_{n-1}^{*2} \eta_n^2 = N^{-1} \sum_{n=1}^{N} x_{n-1}^{*2} \sigma_\eta^2 + o_p^*(1) \tag{1.8}$$

and

$$N^{-1} \sum_{n=1}^{N} x_{n-1}^{*2} \xrightarrow{P^*}_P \frac{\sigma_\eta^2}{1-f^2}, \tag{1.9}$$

to argue that

$$M_N^* \xrightarrow{D^*}_P N\left(0, \frac{\sigma_\eta^4}{1-f^2}\right). \tag{1.10}$$

(c) Finally, use this to argue that $W_N^* \xrightarrow{D^*}_P \chi_1^2$ under H, where

$$W_N^* = (\hat{\phi}_N^* - f)^2 \sum_{n=1}^{N} x_{n-1}^{*2} / \sigma_\eta^2 \tag{1.11}$$

is the bootstrap statistic.

2 Co-integration

Consider a $p = 4$ dimensional vector with the log of end-of-week stock prices, $\{Y_t\}_{t=0}^T$, with

$$Y_t = (A_t, B_t, C_t, D_t)',$$

where A_t, B_t, C_t, and D_t denote the equity prices of large commercial banks with approximately the same risk profile. Simple corporate finance theory suggests that the stock prices for the four banks will be closely linked.

Consider a co-integrated VAR(1) process as given by the error-correction form

$$\Delta Y_t = \Pi Y_{t-1} + \epsilon_t, \quad t = 1, 2, ..., T, \tag{2.1}$$

with $\Delta Y_t = Y_t - Y_{t-1}$, Y_0 is fixed, $\{\epsilon_t\}_{t=1}^T$ i.i.d.N$(0, \sigma_\varepsilon^2)$ distributed and subject to the reduced rank restriction

$$\Pi = \alpha\beta', \quad \text{with} \quad \alpha, \beta \in \mathbb{R}^{4\times 3}. \tag{2.2}$$

#2.1 Pairs-Trading

Due to the similarities between the chosen stocks, it is postulated that

$$\beta = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{2.3}$$

(a) Discuss, how you can use the information in this system to design a pairs-trading strategy.

#2.2 Missing Observations

As above, assume that only $(Y_0, Y_2, Y_4, ..., Y_T)$ are observed while $(Y_1, Y_3, ..., Y_{T-1})$ are unobserved. Set

$$(X_0, X_1, ..., X_n, ..., X_N) = \left(Y_0, Y_2, ..., Y_{t/2}, ..., Y_T\right), \tag{2.4}$$

with $N = T/2$ an integer.

(a) Derive the error-correction equation for the process $\{X_n\}_{n=1}^N$,

$$\Delta X_n = PX_{n-1} + \eta_n, \quad n = 1, 2, ..., N, \tag{2.5}$$

with $\Delta X_n = X_n - X_{n-1}$.

(b) State the co-integration rank, $r_X = \text{rank}(P)$, and set $P = ab'$. Compare with the co-integration rank for the process Y_t, $r = \text{rank}(\Pi)$, and compare a and b with α and β.

(c) State the moving average solution for X_n as a function of $\{\eta_n\}_{n=1}^N$ and the initial value, X_0. In particular present the stochastic trend(s), $\tau_n = \sum_{i=1}^n a'_\perp \eta_i$, and the loading(s) in terms of $b_\perp(a'_\perp b_\perp)^{-1}$. Compare with similar results for Y_t.

#2.3 Empirical Analyses

The data file `Exam_2020.xls` contains weekly observations for the four series $\{Y_t\}_{t=0}^T$ denoted Aw, Bw, Cw, and Dw for $T = 400$ and bi-weekly observations for the process $\{X_n\}_{n=0}^N$ denoted Abw, Bbw, Cbw, and Dbw for $N = 200$.

(a) For each of the time series, $\{Y_t\}_{t=0}^T$ and $\{X_n\}_{n=0}^N$, perform a co-integrated VAR analysis. In particular:

[1] Set up a VAR model for the series and check the model assumptions.

[2] Determine the co-integration rank. Be precise on the test procedure you apply and explain why (or why not) it is preferable to use a bootstrap version of the rank test.

[3] Impose the reduced rank and impose identifying (and potentially also over-identifying) restrictions on β and test the restrictions implied by the structure in (2.3).

[4] State the estimated moving-average solution.

(b) Compare the two sets of results and relate to the results from the theoretical analysis above.

(c) Find periods in your sample, where a pairs-trading strategy would have given a positive return.